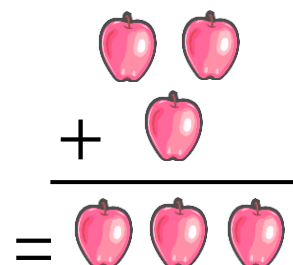

COMBINING LIKE TERMS

You've probably solved some simple equations in your course, equations like $x + 5 = 10$ or $-3n = 20$. These equations have the variable in just one place. What would we do if we came across an equation like $7x - 2x - 9 = 12x + 1$, with that darned variable all over the place? I suppose you can guess that this chapter will give us the tool we need to solve that kind of equation.



□ *THE COMMON-SENSE APPROACH TO COMBINING LIKE TERMS*

If we add 5 inches and 6 inches, we get a sum of 11 inches:

$$5 \text{ inches} + 6 \text{ inches} = \mathbf{11 \text{ inches}}$$

If we subtract \$14 from \$50, we get a difference of \$36:

$$\$50 - \$14 = \mathbf{\$36}$$

If 7 widgets were sold last week and 3 widgets were sold this week, then the two-week total is 10 widgets. We can write

$$7 \text{ widgets} + 3 \text{ widgets} = \mathbf{10 \text{ widgets}}$$

And if we let w represent a widget, we can write this fact as

$$7w + 3w = \mathbf{10w}$$

and we say that $7w$ and $3w$ are *like terms*, because each term represents some number of the exact same item.

Similarly, if 500 widgets were stolen from a warehouse that originally stored 2,000 widgets, we can write the amount remaining as
 $2,000 \text{ widgets} - 500 \text{ widgets} = 1,500 \text{ widgets}$, or

$$2,000w - 500w = \mathbf{1,500w}$$

We can also do this simplifying with more than two terms. For example, if we add 7 apples and 9 apples, then subtract 6 apples, and lastly add 10 additional apples, we can find the final number of apples like this:

$$7A + 9A - 6A + 10A = \mathbf{20A}$$

Homework

1. Work each problem, using the idea of *combining like terms*:

- | | |
|---------------------------|--|
| a. 9 feet + 12 feet | b. 12 lbs – 3 lbs |
| c. 14 apples + 14 apples | d. \$123 + \$234 |
| e. 100 meters – 98 meters | f. 17 widgets + 3 widgets |
| g. 93 cm – 90 cm | h. 3 iPads + 7 iPads |
| i. €17 – €7 (€ = euro) | j. $34 \text{ m}^2 + 16 \text{ m}^2$
(m^2 = square meter) |

❑ **SOME TERMS CANNOT BE COMBINED**

We cannot do this adding (or subtracting) unless the items we’re adding or subtracting are exactly the same. This is the old “*you can’t add apples and oranges*” idea. Consider trying to analyze the sum “4 apples + 1 orange.” Although you may “add” these together and say that the total is 5 pieces of fruit, you must agree that the sum is not 5 apples and it’s not 5 oranges.



We say that “4 apples” and “1 orange” are **unlike terms**; they cannot be added together. The terms $3x$ and $2w$ are also unlike terms and so the sum $3x + 2w$ cannot be simplified. Similarly, $7n$ and 5 are unlike terms and cannot be added together to make a single term.

This “simplification” process (assuming we have like terms) is called **combining like terms** and is used constantly in algebra. It basically says that regardless of what a variable might mean, we can combine two or more terms with the same variable into a single term by merely doing the arithmetic on the numbers connected to the variables; these numbers are called **coefficients**. For example, when we write

$$5u + u = 6u$$

we say that the first term has a *coefficient* of 5, the second term has a *coefficient* of 1 (understood), and the sum (the answer) has a *coefficient* of 6.

□ **EXAMPLES**

A. $10c + 20c = 30c$

B. $9x + x = 9x + 1x = 10x$

C. $-7y + 4y = -3y$

D. $\frac{2}{3}t - \frac{2}{3}t = 0t = 0$

E. $18m - 20m = -2m$

F. $-10h - 3h = -13h$

G. $25a - 13a - a = 11a$

H. $7x + 9y$ cannot be simplified (unlike terms)

I. $13N - 2$ cannot be simplified (unlike terms)

J. $7x - 7 - 9x + 12 = 7x - 9x - 7 + 12 = -2x + 5$

Homework

2. Your study partner is resisting the idea of like terms, and insists that $3x + 7$ ought to equal $10x$. By letting $x = 5$, prove your partner wrong.

3. Simplify each expression by combining like terms:

a. $10n + 7n$	b. $8x - 8x$	c. $14z + 14z$
d. $19a + a$	e. $32r - 20r$	f. $8x + 7y$
g. $77n + 77$	h. $24z - 24z$	i. $12Q - 11Q$
j. $-8x + 5x$	k. $-9t - t$	l. $12a - 20a$
m. $-9 + 9L$	n. $-7e - 7e$	o. $-12k - 13k$

4. Simplify each expression by combining like terms:

a. $3x + 7x - 8$	b. $9 + 2g - g$	c. $7x - 3x + 10$
d. $7 + 5x - 3$	e. $12d - 13d + d$	f. $4 - 7x + 9x$
g. $7y + 3y - 10y$	h. $-x - 2x - 3x$	i. $7w + 2w - 20w$
j. $-6r + 3r - r$	k. $30 - 3q + 3q$	l. $9u - 12 - 12u$

5. Simplify each expression by combining like terms:

a. $7x - 3 + 8x - 5$	b. $17 - n - n - 1$
c. $20q - 8q + 8 - 8$	d. $-1 - x - 1 - x$
e. $40i - 8 - 39i - 8$	f. $4v + 7 - 7v - 34$
g. $33w + 99 - 99 - 33w$	h. $-7x + 6x + 2x - 8 + 8$

Review Problems

6. Simplify by combining like terms:

$$7n + 2n - 8 - 1 + 5n - n - n + 9 - 13n + 20n$$

7. Now your study partner is insisting that adding $3n$ and $2n$ should come out $5n^2$. That is, he claims that $3n + 2n = 5n^2$. Prove him wrong again.

Solutions

1. a. 21 feet b. 9 lbs c. 28 apples d. \$357
 e. 2 meters f. 20 widgets g. 3 cm h. 10 iPads
 i. €10 j. 50 m²

2. We will compare the expression $3x + 7$ with the expression $10x$ specifically when $x = 5$:

$$3x + 7 = 3(\mathbf{5}) + 7 = 15 + 7 = 22$$

$$10x = 10(\mathbf{5}) = 50; \text{ not the same}$$

Therefore, $3x + 7 \neq 10x$. By the way, $3x + 7x$ would equal $10x$.

3. a. $17n$ b. 0 c. $28z$ d. $20a$ e. $12r$ f. As is
 g. As is h. 0 i. Q j. $-3x$ k. $-10t$ l. $-8a$
 m. As is n. $-14e$ o. $-25k$
4. a. $10x - 8$ b. $9 + g$ c. $4x + 10$ d. $5x + 4$ e. 0
 f. $4 + 2x$ g. 0 h. $-6x$ i. $-11w$ j. $-4r$
 k. 30 l. $-3u - 12$

5. a. $15x - 8$ b. $16 - 2n$ c. $12q$ d. $-2 - 2x$
 e. $i - 16$ f. $-3v - 27$ g. 0 h. x

6. $19n$

7. We'll show him the error of his ways with a numerical counterexample; we'll let $n = 10$. On the one hand,

$$3n + 2n = 3(\mathbf{10}) + 2(\mathbf{10}) = 30 + 20 = 50.$$

On the other hand,

$$5n^2 = 5(\mathbf{10}^2) = 5(100) = 500.$$

Hence, $3n + 2n \neq 5n^2$.

P.S. Get a new study partner!

*“In the middle of
difficulty
lies opportunity.”*

--Albert Einstein (1879-1955)